RUSSELL'S BARBER

AND

OCKHAM'S RAZOR

There would seem to be a predestined metaphorical use of Ockham's celebrated Razor in the hands of Russell's equally celebrated Barber.

The author's Thesis of Self-Other Existential Relativity provides the philosophical basis for a consistent Neo-Cantorian Theory of Sets for Mathematical Logic.

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Queen of the South Press

This Exercise in Linguistic Analysis has been upgraded from Chapters 6 to 11 of the author's book

- ACHIEVING THE IMPOSSIBLE -

The Quest of Science for the Self of the Cosmos.

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INTRODUCTION

Peter Lock was born in Adelaide on August 10th. 1923. He has University degrees in Pharmacy, Science and Theology and has taught a wide variety of subjects from Religious Education and English Expression to Physics, Chemistry and Mathematics. For many years he was an active member of both the Science Teachers' Association and also the Mathematics Teachers' Association of Victoria. For a short period he lectured at Tertiary level on the teaching of The New Mathematics. He lives now in retirement in Adelaide

The author claims to have achieved what was generally deemed impossible. For over a Century the best mathematical minds in Academia have sought to remove the trivial but frustrating Paradoxes teasing the foundations and the formal structuring of a simple Theory of Sets in Mathematical Logic. Experts concluded it could not be done, due to the inadequacy of language to cope with the seeming contradictions of self-reference and not self-reference situations.

This writer's introduction of the simple logic of self-other Existential Relativity into the foundations of Mathematical Logic not only disposes of the troublesome paradoxes in Cantorian Set Theory, but validates some of Hilbert's intuitions and his proposed formalism in structuring the foundations of Mathematics. With simple linguistic analysis, his thesis demonstrates an intrinsic flaw in Gödel's Proof and renders its method inadequate and its conclusions untenable. It initiates a new approach with respect to consistency in Mathematics and raises completely new issues for, and extends the horizons of Theology, Science, Cosmology, Ecology, Economics, Sociology and Psychology in cultural evolution.

DISTINCTION AND UNION

Growth can only take place through the union, as a whole, of increasing distinction of parts. It is expedient to reflect on the logic of both distinction and union, on *and* and *or* and on *and-or*. These are language's simplest joining words and the implications of their meaning easily elude us.

Two distinct statements can be joined by the word *and* to form one composite statement which is called the conjunction of the original separate two. Jack rows the boat. Jill also rows the boat. Both Jack rows the boat and Jill rows the boat. The use of *or* is not quite so simple.

We can say that either Jack rows the boat or Jill rows the boat or both Jack rows the boat and Jill rows the boat. We can dispense with the formal sentencing and economize on words by simply stating that either Jack or Jill, or both Jack and Jill row the boat. We now have with the use of *or* what is called the disjunction of the original two statements.

In this example of Jack and Jill we have the disjunctive *or* used twice, first to separate by distinguishing Jack from Jill, and then to separate their *or*-disjunction from their *and*-conjunction. The conjunction effected by *and* may be simply the logical joining of two ideas, one after the other, or it may verbally refer to processes which have profound psychological and physical overtones when the parts are also welded together into a whole. In Chemistry, both *or* and *and* are involved in mixtures, but a special ordered sort of unitive *and* is the linkage in chemical compounds. Psychically our minds are full of mixtures of *I* and-or *you*. Willpower's psychic energy compounds them into *we*, *us*, *ours*.

Such meaning given to *or* whereby there is implicitly included the possibility of a further *or* linking the first *either...or...*with also *both...and...*is often referred to as the inclusive *or*, and would be translated into Latin by *vel*. The English, *either* this *or* that, implicitly including, *or* both, would be rendered by *vel...vel...*The distinctioning Latin mind also considered an *aut...aut...*which is still given in English as, *either...or...*but excludes the additional alternative of *both...and...* This is the mutually exclusive *or*, as when we say that the letters of the alphabet fall into categories of either consonants or vowels. As a whole, the alphabet is made up of both vowels and consonants. By analysis in its parts, each letter is either a

vowel or a consonant and cannot be both a vowel and a consonant. The letter *y* has consonantal value and also serves as a possible substitute for vocalic *i* in any position.

The extended analysis of the inclusive *or* to take into actual account the alternative *or both...and...*is associated with positive and integrated thinking, whilst the exclusive *or* is linked with the negative, differentiated, *this and not that*. The literary pun derives its effect from a word or phrase having both one meaning and also an other. The mind then enjoys its choice of one meaning, or another meaning, or both meanings together.

In the Set Theory of Modern Mathematics, if one existing set of circumstances mutually excludes another set of circumstances, the two sets are said to be disjoint. Very often two disjoint sets can complement each other to form a whole or universal set, as we have seen with the vowels complementing the consonants to effect in their union the universal set of letters of the alphabet. When two sets are such that they are both disjoint and also complementary in their union making the whole or universal set of discourse, then one is said to be the *notset* of the other. When all that we are discoursing about is the set of letters of the alphabet, the set of not-vowels is the set of consonants and the set of not-consonants is the set of vowels.

There is no great difficulty in understanding this kind of negation which, although it uses the word *not*, does not imply the sense of contradiction but of complementarity indistinction. In their distinction and union to form now an *all* of discourse, *self* and *other* are both disjoint and complementary, and hence the ***notself*** becomes synonymous with the ***other*** and vice versa. As long as *notself* is rendered by *other* no problems arise, but to try to positively self-relate terms involving and expressed with the words, **self** and **notself** and **not self,** is to become confronted inevitably with self-contradictions. *Notself* (one word) is the true and proper complement of *self*, and is synonymous with its *otherself*, or just plain *other*. *Not self* (two words), by definition, means neither *self* nor *otherself*. In its complete self-negation, it contradicts both *self* and *otherself*.

By not explicitly identifying the ***notself*** with the ***other*,** one aspect of the linguistic resolution of the self-reference paradoxes in logic has been held up for over a Century. Commentators who still

persist in using the self-negated reflexive *not self* have necessarily become trapped with pernicious paradoxes or seeming contradictions. There is only one literal self-contradictory term in English and that is *not self*. If the true and proper *not-self* exists, it exists in an *other-self*. *Not self* denies the real existence, in any way, of *self* or *other-self* and therefore denies any form of ***self-existence*.**

Sometimes the exclusive disjunction *or* links what are generally and loosely thought of being opposites or contraries, such as life or death, true or false, good or bad, yes or no, some or none. We need to use a cautious discernment whenever invoking the Principle of Contradiction which accepts the simple alternative of either an assertion or its proper negation being what they are, but not their conjunction. "Either *this is* or *this is not*, but *this cannot both be, and also not be, under the same set of circumstances*."

Often we use the word *opposite* when we should be more precise and employ another word like *other* itself. We speak glibly of the opposite sex when we should refer to it as the other sex.

The existence of true free choice and its ultimate perfection demands situations which admit a quaternity of alternatives, as in, **OR** neither self nor other, **OR** either self **OR** other in distinction, **OR** both self and other in distinction and union. We link free will with the act of choosing between one thing and another. This is not a correct understanding of free will. Animals make choices about events whose outcome results from their brains' complex *If... then...* foresighted programming. Their act of choosing may not be conscious as in humans, but our conscious choosing merely follows on from the intuitive awareness of the existential relation of the self's judgment and conscious identification of itself with one thing, or another, or neither, or both things at once. Free will is not an action, but a self-orientating existential relation in the quaternity of alternatives - OR neither, OR this, OR that, OR both this and that. Readers acquainted with Topology or Rubber Sheet Geometry as it is called, will already be aware of this quaternity in the Four Colour Theorem associated with coloured maps and the like. The Theorem states that only four distinct colours are needed to separate effectively any number of countries on a map, providing their defined boundaries are linear and not point discontinuities.

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Because our English *or* is equivocal in its distinctioning, being either implicitly inclusive or exclusive of the alternative conjoining *and*, it is convenient to make use of *vel* and *aut* to make distinctioning more distinct. Henceforth, we shall refer to the inclusive *and-or* as VEL and the exclusive *or* as AUT. VEL promotes hilarity in its triple doublets, namely, OR, *or this or that*, OR, *both this and that*. AUT is depressing and is limiting because one cannot both have one's cake and also eat it. VEL can elate but AUT can frustrate. With VEL, one has a choice of two other choices. Free will or choice is perfected in relations involving VEL, whilst it loses a degree of freedom in AUT relations. AUT is the basis of pure analysis and is just concerned with separating the bare parts. VEL combines both analysis and synthesis, and if some unity eventuates, the whole becomes more than just a disordered junk collection of once separated bits. VEL both adds and multiplies. AUT subtracts or divides. With *or* the self experiences true separation, with *and* things are brought together. There are quite different kinds of unions involving togetherness.

There may be a simple spatial arrangement of things in close physical proximity like persons crowded in an overloaded bus. The crowd, as a simple accumulation of people, is no more than the sum of the people. A union resulting in a functioning whole becomes a unity that is qualitatively more than the mere accumulation of its separate conjoining parts. There is a very meaningful analogy between the compounding together of chemicals and atomic particles in the physical realm and the compounding of words and concepts describing our experiences in the psychical. If favourable pre-union energy conditions prevail, one heavy hydrogen or deuterium nucleus can be made to fuse together with an other deuterium nucleus to form one new nucleus of helium. A one and a one become an other one. As regards the number of independent entities, the reaction in quantity is one less, but in quality one more. **TWO ONES BECOME ONE TWO**.

Psychically we witness an homologous patterning. We make or name a *one* and a *one* to become, *a one-plus-one-oneness of ones*. Such an added oneness of *ones* becomes a *one* of twos. The union of distinction's this *one* and that *one* effects the unity of *one's* new two which is actually a biunity or a two-in-one. Since there were two

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distinct *ones* to start with and a new *one* proceeded from their distinction-in-union, the addition process now as a whole, really involved three *ones* in unity, a triunity or trinity.

The personal affective metamorphosis when "*I*" and "*you*" become "*we*", and when "*mine*" and "*yours*" become "*ours*", both conceptually and perceptually, is brought about by a fusioning nuclear-self implosion. Reflexive self-functioning "*I-me*", through its transitive other-dependent "*you*", grows to maturity as integrated "*we-us*", from a singular unit to a plural unity, from a part in an AUT to the whole of a VEL.

Unity as a pure abstraction is the very essence of *oneness*. In the reality of existential relativity, unity is the union of all of distinction's relational states of distinction and union themselves. The essence of union requires the existence of distinction. Holistic thinking contemplates the parts together now as a real whole - it sees both the parts and the whole in union, as a single entity or unity.

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THE ONE AND THE ALL

The ability to count with ordered numbers represents an activity which is uniquely human and which is at the very essence of our psychic self-life existence. The one, the some, the many, the all, the infinite are abstractions made by conscious reflection on and in actual sensible experience. Mathematics is a manifestation of intelligence. The Greek word from which Mathematics is derived means to learn by human enquiry. Mathematicians study relations as relations. For them it is nowise necessary to name specifically the things that are related, but only to have in mind the general type of connection existing between them. This is the rationale behind Bertrand Russell's well-known facetious epigram that Pure Mathematics is "the subject in which we never know what we are talking about, nor whether what we are saying is true."

Mathematicians have taken a long time to find out what they think they are talking about when they speak about numbers, and still do not know why their reasoning does not ring true in some linguistic problems associated with the basic logic of Set Theory.

In the physical world, two apples plus three apples equals five apples. This is external quantitative addition. In the psychical inner realm of qualitative forms and figures, one 2 added to one 3 makes one 5. Psychically a one and a one add up to a further one. One polyunity added to one other polyunity adds up to one further new polyunity. In binary positional arithmetic which is the basic baby language for all computation in electronic computers or calculators and probably the human cerebral computer as well, there are only two digital signs needed, 1 and 0. These provide the two elements for functional distinction whilst their union in different combinations effects the required numeral. In binary, there are no single special signs for twos or threes or more of things, but only their names. At one and the same time, one's self can become any named collection of other ones, both in distinction and also in union. The name of this experienced psychic becomingness is called a number.

Today, after a long evolutionary process we are able to give names to many different kinds of number-experience. The very simplest are the positive integers or whole numbers 1, 2, 3,... There are also negative integers -1, -2, -3,...and there is zero 0. The set of the positive and negative integers, together with zero, constitute the Set of Integers. The Set of Positive Integers is also known simply as the

Set of Natural Numbers. The Set of Positive Integers with the addition now of zero is generally called the Set of Cardinal Numbers, since answering the question *How Many*? hinges on their use. The Latin word *cardo* means hinge.

The Set of Integers is a subset of the Set of Rational Numbers or Fractions. Rational refers to the ratios of parts to a whole. The whole numbers or integers can be considered as fractions with denominator 1. Many real number experiences, like the measure of the length of the diagonal of a unit square, are not able to be expressed as a simple ratio of two whole numbers and are said to be irrational, i.e. not able to be made a ratio. Such numbers like the square root of 2 or the cube root of 3 are members of the Set of Irrational Numbers. The Set of Real Numbers is made up from the union of the Sets of the Rational and Irrational Numbers. There are still other types of numerical experience and these give rise to transcendental and complex numbers, of which more will be said later when treating of Unity and Infinity.

Strictly speaking, 0 and 1, called zero and one, are numerals or signs representing numbers. The decimal ten-fingered notation makes use of ten different signs 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and positions them according to powers of ten. In binary, we make use of the minimum number to afford distinction, that is, just two, 0 and 1, and position these according to powers of two, as in 1 + 1 = 1 0. This is not ten, but one, zero, and means 1 two and 0 units. 1 + 1 + 1 = 1 1, which is read simply as one, one, and means 1 two and 1 unit. In conventional decimal notation where there are ten digital signs, 1 + 2 + 3 + 4 = 10, that is, 1 ten and 0 unit. In binary notation, it becomes an unwieldy 1 + 1 0 + 1 1 + 1 0 0 = 1 0 1 0 and read as one, zero, one, zero, that is, 1 eight and 0 four and 1 two and 0 unit. Binary is cumbersome for ordinary human activities. Decimal is far more efficient and adapted to our abilities. Binary is admirable for electronic circuitry where only the duality, + or - , on or off, up or down, which are linked with dipolar phenomena, can then be utilized. Binary adds extra meaning to human thought processes, and both philosophically and psychologically is very satisfying and revealing.

Philosophically, Number Theory begins with the distinction between something and nothing, between a *one* and *none*, between

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THE ONE and THE NONE, between 1 and 0. At the basis of all numbering, there is only the unit number ONE, which has its being in *oneself's* ONE self. All other numbers are unities, polyunities, many-ones-in-one, onenesses of other ones as yet an other one. By psychic ovoidal sense-linked fission, oneself's spaced time ONE becomes two ONES, and the two ONES then, by cognition's fusion, become ONE two. ONE exists. NONE does not exist of its own, but only as oneself's NO OTHER-ONE.

We conceive empty rooms and void spaces, but factually a room can only be said to be verifiably empty, or a space void, when there is no one other than the observer in it. A perceived void is really an ovoid with only one focus in use like the Sun in the planets' elliptical orbits. The NONE is not a negation or contradiction of ONE, like NOT ONE, but rather a complementary NOT-ONE, a NO- OTHER-ONE but oneself's very own ONE.

In the notation of binary arithmetic, ONE signed as 1, is now fertilized by its complement, the NOT-ONE or the NONE now signed with zero's 0, and through their positional union, a new complete system of numerals is brought creatively into existence. Creation of something itself, out of a nothing-of-itself, only makes sense when the nothing-of-itself is considered as an existing *self's* complementary *otherself*. This latter now fertilizes the pre-existing something of itself, and by their union, effects a new *self* and *notself* unity, a *self-other* unity.

Counting today involves an ordered succession of numbers with each one, one more than its predecessor and the sequence starting usually with zero. The primitive answering of the question "How many?" generally entailed a matching process, a simple one-to-one correspondence, as for example, between the known name-number of stones in a bag or the notches on a stick and the number of goats in the herd. Such one-to-one correspondence might also be between a named number of fingers and the members of a family around a hearth. Today, two distinct collections having the same number of elements are said to be equivalent.

The question "How many?" can be given a precisioned answer with a number. However, to define *number* as being the answer to such a question is merely begging the question. What is a number in itself? Language, as true self-other-communication, speaks for itself,

for its dictator self. A number only exists in a self. We give names to numerical experiences of spaced things. Intuitively on recollection, my own unique inner reflexive awareness reminds me that I am a ONE, and that all around me there are other ones. The experience of finding an empty room, an empty stable, an empty glass, initiates my use now of no one, or none. I am sensibly aware of one leg and of one arm on one side of my one body, and of one other leg and of one other arm on one other side of my one same body. Having psychically identified my one self with both at once, I am able now to experience something numerically new and I name such experiences *twos* of things. The same applies to names of collections of threes and fours, or more, of things. Ones of things are units, twos of things are biunities or sets of two-in-one, threes of things are triunities or sets of three-in-one, and numbers of things are polyunities, or sets of many-in-one. Numbers are no different from universal ideas. They are universal ideas, which are collections of all collections of special things.

If instead of the word *collection*, we employ another word *set*, then a number, being a universal idea, is a species of *set of all sets.* This is the essence of the definition of *number* which Frege and following him, Russell came to at the turn of last century.

In Russell's own terms, the number of a class was the class of all those classes that are similar to it. We may just as well say that the number of a set is the ascribed name of the set of all those sets that are equivalent to it. Equivalent means giving the same answer to the question "How many?" Today, we interpret *similar* in Set Theory to imply *equivalent* with the added property of order. Russell chose to use the word *class.* Others used the words, *set, family,* and the like. Whatever word we use, they are types of a grammatical entity called a collective noun, singular in form and paradoxically plural in meaning. That branch of *New Mathematics* which concerns itself with the relational nature of these entities is called Set Theory, the rudiments of which are taught to quite young children in primary school, and linked to their understanding of *and*, and of *or*, and of *and-or.*

When strict logical investigation was applied to Set Theory and its axiomatic formulations, seeming contradictions began to raise their teasing heads, and left the initial foundations of Mathematical Logic

in an unsettled state from which it recovered, but only by circumventing the paradoxes, not removing them.

An Algebra of Sets was introduced by Georg Cantor, between the years 1871 and 1884, in his attempts to solve problems involving trigonometric series. In it he developed, among other things, a theory of cardinality of sets, which initiated new approaches to the theory of infinity. The notion of a set was that of a well-defined unique collection, list or class of distinct objects or things. The number of distinct elements in the set was named its cardinality, which was represented by the numerals 0, 1, 2, 3, and so on. These denoted cardinal numbers. The elements of the set, though necessarily distinct, were made to exhibit a togetherness or oneness by being enclosed in the convention of a pair of bracket-like braces { }.

For this writer a more meaningful and precise definition of a set would be, *a well-defined unity of distinct units in intentional union,* a oneness of distinct ones now as a one-continuum. The philosophical basis of Set Theory, as of all Mathematics and Science, is the duality of distinction and union.

The individuality of a set can be made manifest by using a capital letter to designate it. The set of natural numbers can be denoted by N, which then stands for {1, 2, 3,...}. The set of the letters of the English alphabet could be denoted by, say A, which as a sign, stands for {a, b, c,...x, y, z}. How many elements are there in set A? Its cardinality is 26. Any other set too with 26 elements would have the same cardinality, and the two sets would be said to be equivalent. Set A is also said to be finite, since in counting the number of distinct elements in the set the actual counting process comes to an end. There is a discrete specific number of elements in the finite set A.

What about the set N of natural numbers? The counting process of finding out its cardinality never ever comes to an end. Intuitively, we would call this type of set *infinite.* However, there is very much more to infinite sets than this.

Sets have subsets which are well-defined sets of some specific and distinct members of the original set. The set of letters of the English alphabet has the subsets of five vowels and twenty one consonants. With finite sets, the cardinality of the set as a whole must always be greater than that of any proper subset of itself. With ordinary finite physical things, the whole is always greater than the

part. The set of natural numbers can be understood as being composed of the subsets of both the odd and the even numbers. When we try to count the members of these two latter subsets by matching them in a one-to-one correspondence with the elements of the set of natural numbers, we find that they too have the very same cardinality and hence are equivalent. The set of natural numbers is an infinite set; the set of even natural numbers is also an infinite set; the set of odd natural numbers is likewise an infinite set. A set is infinite, if it is equivalent to any proper subset of itself. With infinity, the whole is *no greater* than half of itself or any fractional part of itself

Sets are made up from distinct elements or units. Sets themselves have subsets, which are not the original set's elements in distinction but individual sets of some of its elements in union. There are also sets of sets. The set of all possible subsets of any set is called a power set. With the inclusion now of the word *all,* complications arise from the unlimited use of *all* in the concept of a Set of All Sets. Granted that a Set of All Sets is a power set, there arises the problem whether such a Set is an element itself of this Set of All Sets. If X stands for the Set of All Sets, is X an element of X? Does this new Set now as the Set of All Sets, add one more set to an already existing All Sets? A consistent answer requires a simple distinctioning of *All* into the *Self* and the *Other* of Existential Relativity.

Cantor observed that the set of rational numbers was equivalent to the set of integers, the latter being a proper subset of the former. Hence each set was as infinite as the other. Both would have the same cardinality which he signed as the first letter of the Hebrew alphabet, aleph with the subscript zero, $\pm o$. If an infinite set of objects could be arranged in a sequence similar to that of the positive integers, it was said to be denumerable. However, the set of all real numbers, which includes both rational and irrational numbers, could be shown to be non-denumerable and hence represented a higher type of infinity than that of the rational numbers or the integers. This non-denumerability of the number continuum demanded a higher type of cardinality which he named c.

Cantor recognized that the cardinality $\pm o$ of the infinite set of rational numbers was conceptually different from the cardinality $2^{\pm o}$ of the infinite set of real numbers or from c of the real number

continuum where $c = 2^{\pm o}$. The issue became confused by logicians not recognising that The Set of All Sets X, must be a unique power set, which contained two proper subset units, firstly and reflexively its own unique whole or X-Set itself, and then also transitively the Set of All Other Sets Y, where $X \eta \{ X, Y\}$. In determining the cardinality of the power set of any set whose cardinality is $\pm o$, each element may be considered for selection or for non-selection, i.e., in 2 possible ways. If the number of the units is $\pm o$ the selecting can be performed in $2^{\pm o}$ ways.

Cantor's theory of aggregates and transfinite numbers was his most original achievement and was created almost entirely from his own intuitions. Some people, both then and now, regard his theory as one of the most beautiful and profoundly meaningful of all mathematical creations. Others, frustrated and disturbed by the seeming contradictions inherent in his reasoning, reject his importance. The severe criticism, both personal and academic, of some of his peers and the ignoring of his work by others was too much for his limited self-confidence. He died in a Mental Hospital.

Mathematicians have always striven to put their subject on strict logical foundations. Over a period of years towards the end of last century, Gottlob Frege wrote a two-volumed treatise on the foundations of arithmetic. In this he made free use of the concept of a class of all classes that have some common property. Just before the second volume was published in 1903, he received a communication from Bertrand Russell setting out what is known now in a popular form, as Russell's Barber Paradox. If logically correct, it threatened to undermine some of Frege's main ideas. The latter author made an acknowledgment at the end of his second volume as follows. "A scientist can hardly meet with anything more undesirable than to have the foundations give way just as the work is finished. In this position, I was put by a letter from Mr. Bertrand Russell as the work was nearly through the press."

REMOVING THE PARADOXES

Russell's Barber Paradox proceeds as follows. In a certain village there is a barber who shaves all and only those who do not shave themselves. The self-deceptive question has always been, Who, if any, shaves the barber? If the barber does not shave himself, then he becomes a member of the set of people whom he does shave. If he does shave himself, he contradicts his role which is to shave all and only those who do not shave themselves. A seeming contradiction or paradox rears its annoying head.

In its more formal statement, the Paradox refers to sets of all sets which either do, or do not, contain themselves as members. Let S stand for the set of all sets which do contain themselves also as members, and let T stand for the set of all sets which do not contain themselves also as members. Now T as a set of all sets, is either a member of S or a member of T. If it is a member of S, which is the set of all sets which contain themselves as members, then it contradicts its standing for the set of all sets which do not contain themselves. If T is a member of T itself, again it contradicts its standing for the set of all sets which do not contain themselves, and hence it should belong to S, the set of all sets which do contain themselves.

There are many variations of paradoxes on this theme and some of them will be dealt with in what follows. Commentators at times refer to sets like the above T, which do not contain themselves as members, as normal or ordinary and those like S, which do contain themselves as members, as being non-normal or extraordinary. Implicitly or explicitly they all contain the few words which are the root cause of all the trouble, namely, the universal *all* and the enigmatic, *not self* as distinct in meaning from the one-worded *notself.*

Does *not self-containing* mean *notself containing* or *not any kind of self-containing?* As coined and defined above, *notself containing* is taken here to mean the complementary and transitive *otherself containing,* whilst the second alternative excludes both *self* and *otherself.* Thus the now negated reflexive use of *containing* becomes a negated transitive use as well. Any not self-containing container that does not contain something or **other,** at least potentially is meaningless and lacks all real existence outside the mind. It would appear that Russell, and all other logicians since him, never averted

to an explicit transitive use of ***other*** in the place of the negated reflexive *notself.* Gödel proved its actual necessary use *outside* sets or systems, but failed to perceive the *selfotherness* of existential relativity *within* the universal **SET OF ALL SETS.** If Gödel had recognized the grammatical distinction between operation-verbs used reflexively and-or the same verbs used transitively, he would never have written in the way that he did, his famous thesis on Russell's *Undecidable Propositions of Principia Mathematica.* A proposition in a consistent system is said to be undecidable if neither it nor its negation is provable. There is no systematic way of deciding whether a proposition is a *not self* negation or a *not-self* complement with respect to negated self-reference, unless we make the prior distinction of operation-verbs used reflexively and-or transitively.

There is a principle in Philosophy known as Ockham's Razor. Complicated explanations should not be invented or continued in use, if more satisfactory simple ones exist or can be found. For this reason we prefer the Copernican system to the Ptolemaic in describing planetary motion. There would seem to be a predestined metaphorical use of this celebrated Razor in the hands of Russell's equally celebrated Barber. All the linguistic contradictions of Set Theory in Mathematical Logic can be removed by the addition of just one word, or by the mere elimination of just one word and its replacement now by another word.

The new word to be added or used in substitution is *other* or *others.* The word requiring to be eliminated and then replaced, is *notself* or its plural, *notselves*, or their grammatical equivalent. *Notself* can be the complementary *otherself. Not self* does not in any way exist of its own, but only as the mind's concept of the complete contradiction of all self. If the *notself* really exists, then it exists only in an *otherself.* In the existential relativity of knowledge, the alpha-self becomes psychically identified with its real notself or otherself, its omega-self at the ovoid self's other focus.

Set Theory takes cognizance of all this. As with the set of vowels and the set of consonants, one set is the complement of the other and is called the notset or the notself of the other. The *all* may be considered as the unity resulting from the union of one subset itself and such set's complement, which is synonymous with its notset, i.e., its notself or otherself. The *All* exists inside the mind of an alpha-self

who identifies its omega-self with *all other* to form contemplation's unique experience of unity or oneness. Conscious knowledge is a self's experience of containing both its own reflexive alpha-self's being and also its spaced time transitive omega-self's becomingness.

There is a **L**aw **O**f **G**eneration **O**f **S**ets which applies universally to *all-sets.* It is simply that *self's* **all** is the unity effected by the union of distinction's **self** and **all other.** For economy of words we need only take its first letters, and with acronymic succinctness, call it the **LOGOS Rule.** It turns *all* into a sharp two-edged sword of self and other and its cut lays open the very existence and essence of all knowledge and psychic activity in Existential Relativity.

The linguistic salvation of the logical foundations of Set Theory can be achieved by the introduction of the simple concept of the *other.* The negative or non-reflexive *notself* is very often employed in Modern Set Theory, but to this writer's knowledge it has yet to be explicitly identified, linguistically, logically and systematically, with the positive and transitive *other.*

Linguistic analysis makes evident that operation-verbs, where both a subject and an object are involved, can be used reflexively or transitively or both. If used reflexively, then the object of the action becomes also the subject itself. *The barber shaves himself.* If not used reflexively, then the action of the subject must flow across to something or somebody other than the subject itself, i.e., it is used transitively. *The barber shaves other people.* Of course, the barber may shave both himself reflexively, and also shave other people transitively. Those people who do not shave themselves reflexively, yet who do get themselves shaved, must of necessity be transitively other-shaved, i.e., shaved by an other.

Logically, there are three distinct sets of barbers' shaving razors, self-shaving only, others-shaving only, and both self-and-others-shaving. Russell's Barber either has no facial hair to shave or sports a beard, since he professedly only uses others-shaving razors. He now advertises himself as one who shaves all and only those who are other-shaved**,** i.e., those who, not being reflexively self-shaved, are shaved transitively by an other. What now of Russell's original formal paradox concerning the Class of All Classes which do not contain themselves as elements? Some logical distinctioning should

resolve this problem. A well-defined Class of All Classes can be understood as made up of three possible subclasses.

Firstly, there is the class of all classes which contain as their distinct units or elements both themselves and also other classes as well.

Secondly, there is the class of all classes which contain as their distinct units or elements themselves only and not other classes.

Thirdly, there is the class of all classes which contain as their distinct units or elements other classes only and not themselves.

The only class which satisfies the given requirement of being a class of all classes not containing themselves as elements is the third which contains as elements other classes only.

For those who may prefer to use the terminology referred to above, of normal or ordinary sets of sets which do not contain themselves as members, and non-normal or extraordinary sets of sets which do contain themselves as members, all contradictions now vanish, when normal or ordinary sets are considered logically and precisely as sets which contain other sets only.

This argument may be plausible, and logically consistent and satisfying, but it still leaves much to be desired. In what sense, other than for a self-functioning-feedback-system, can a set be said to contain itself as an element, or not to contain itself as an element? In what sense can a unity of units be said to contain itself as one of its own units? The set of all finite sets is an infinite set and hence does not contain itself as one of its own elements or own units. What of the set of all infinite sets? What of the set of all both finite and infinite sets? What of the proscribed *Set of All Sets?*

Before continuing with this theme, which requires another chapter for its elucidation, it is convenient and instructive to take up here the two-edged sword of the LOGOS Rule, the Law Of Generation Of Sets, where self's ALL η {Self, All Other-self} and do battle with the dragon which has plagued Logic right from the times of Epimenides.

Epimenides was a Cretan. He is purported to have said that "All Cretans are liars." If *all* as spoken by a Cretan excludes *self,* as indeed it must if Epimenides is to retain or merit any credibility, then his statement should be explicit in the form, "All other Cretans are

liars" and should be implicitly understood as "I, myself, a Cretan, am not a liar, but all other Cretans are liars."

A similar line of reasoning disposes of those graffiti which advocate "Ban all graffiti." They should really state "Ban all other graffiti." To include itself is to be committed to suicide. We may carry this approach to some generalizations which can benefit from the LOGOS Rule. The author only vouches for the formal logic of these statements being consistent and does not guarantee the necessary correctness of their material logic. Such general statements are, e.g., There is only one absolute rule and that says that all other rules are relative. There is only one rule that has no exceptions, and it says that all other rules have exceptions. There is only one rule that cannot be doubted, and it rules that all other rules can be doubted.

There are more variants on the Barber theme of self-reference and not self-reference and their resolution is instructive in as much as it confirms all that has already been said. Grelling's Paradox would divide the set of all adjectives into the union of those which are self-descriptive or autological, and those which are not self-descriptive or heterological. *English* is English, whilst *German* is not German. *Pentasyllabic* is pentasyllabic, but *monosyllabic* is not monosyllabic. To what set does the adjective *heterological* itself belong, the subset heterological or the subset autological? If *heterological* is heterological, then it is being self-descriptive and becomes autological. If *heterological* is autological, it must then become self-descriptive and so contradicts its being, by definition, not self-descriptive.

We have to lay aside again Russell's epigrammatic admission that in Pure Mathematics we never know what we are talking about, and indulge in some material logic. As observed above, *notself* (one word) is taken to mean some *otherself. Not self*, (two words), means neither *self* nor *otherself.* What does *not self-descriptive* mean? Does it mean *notself descriptive,* or does it mean descript*ive of not self.* The former must be accepted since the latter rules both its *self* out and all *otherself* as well. Autological means self-descriptive. Heterological means others-descriptive. Grelling's universal set of adjectives can be considered as the union of those which are self-descriptive or autological and those which are descriptive of others or heterological. Anyone who asks whether *heterological* is

heterological or autological, does not really know what they are talking about. Heterological, as meaning heterological, means applying only to others and has absolutely no real meaning applied to itself. By definition, *heterological* applies to all other words and excludes itself. People who ask whether a not-square figure is square betray their ignorance of what constitutes a square.

We have found an other-container for the Set of All Sets that do not, or could not, contain themselves. We could have avoided any such contradictions in the first place by precisioning such a unity now as the Set of All Other Sets that do not contain themselves as elements. If Cantor had considered Y as standing for the Set of All Other Sets, then neither he nor others would ever have had to worry about whether Y adds to its very own cardinality or not, namely, $2^{\pm o}$ - 1. Russell's Barber Paradox could also now be rephrased to be made consistent in an other way when it is understood that professedly and in point of fact, the barber in question, with or without a beard, shaves all and only those **others** who do not shave themselves.

Prior to Russell's problems at the turn of the previous century, there had been voiced the Burali-Forti Paradox. It arose because of its reference to Sets of All Ordinal Numbers, whereas it should have been limited just to Sets of All **Other** Ordinal Numbers. An ordinal number can be defined as the Set of All Other Ordinal Numbers which precede it. Zero 0, is taken logically as the first ordinal number. The ordinal number { 0, 1, 2 } is 3.

With other variants on the Barber theme, like Berry's Paradox, we can safeguard our threatened sanity by the appropriate use of the word *other*. Most commentators on the Paradoxes of Set Theory include some presentation of this rather strange exercise in mental gymnastics. For Berry, integers can be represented by numerals 1, 2, and so on. They can also be described in words, e.g., eleven is "the next prime number after seven." It is clear that to express numbers in this way a certain number of words is necessary. We can divide the set of all integers expressible in words into those which can be expressed in less than thirteen words, and those which require thirteen or more words. "The smallest integer that cannot be expressed in less than thirteen words" contains only twelve words, as is evident from counting the words between the quotation marks. We can dispose of the difficulty of this self and-or notself inclusion

simply by rewriting the integer in question as "The smallest **other** integer that cannot be expressed in less than fourteen words."

If Frege had only added one *other* word to his two volumes, he would never have had to make any undesirable acknowledgments. Nor would Ockham's Razor have found such a fertile harvest for its cutting edge. There would have been no compelling motive for Russell and Whitehead to write their *Principia Mathematica* in the way they did. Thus there would have been no need for Gödel's famous Proof and its complicated mathematical approach to consistency and completeness. The latter but echoes the simplest and yet the most profound observation that any scientist can ever make of Nature, namely, that in Cosmology's fundamental Law of Self-Other Existential Relativity, **All growth and subsequent sustainability in any evolved or still evolving system of the Cosmos is reflexively self-functioning and at the same time transitively other-dependent**.

Existential Relativity succeeds where all attempts by logicians to resolve verbally the negated self-reference contradictions have proved futile. Some mathematicians blame the inadequacies of linguistic expression and get around the seeming contradictions by confining their attention to limited formal mathematical languages from which the self-reference situations are conveniently excluded. Russell spent several years trying to extricate his reasoning from this *not self-containing* impasse in Set Theory. It was to no avail. He had to be content to propose with Whitehead, a Theory of (ramified) Types in their *Principia Mathematica*. In their proposed ascending hierarchy, no set could contain itself since it would thus have to belong to a type higher than its own type. Though it might seem to have rid formal set theory of its paradoxes it was only at the expense of introducing artificial stratifications and disallowing certain kinds of sets. Self-reference paradoxes in verbal logic were not resolved, but only expediently ignored.

Many people from other mathematical schools of thought have been far from happy with the situation and using various restrictive formal axiomatic systems, have circumvented the paradoxes without actually removing them. Amongst such are the Zermelo-Fraenkel and the Neumann- Bernays-Gödel formulations. In the Z-F theory there are certain primitive undefined concepts, like the notion of *set*

and *set membership.* Hence one does not try to say what a set is, but in any model, sets are recognized as entities which satisfy the axioms. This is analogous to modern formulations of Euclidean geometry in which *point* and *line* are undefined concepts. By excluding the possibility of any affirmed or negated self-reference, they are made to manifest inner consistency, whilst still leaving the original self-reference logic paradoxes unresolved meaningfully. Millions of words have been written trying to provide for mathematical reasoning in set theory such a logical basis so as to make its conclusions consistent and free from all possible contradictions. Their limited success has been adequate for certain restrictive sets of situations and are not generally applicable to all that modern holistic thinking would intuitively expect.

Most experts in Mathematical Logic are quite satisfied with the status quo. For them, the early logic antinomies are but the former annoying, and now ignored trivia of the past, and there is no real need now for any ***other*** theory which resurrects them even though it may offer a novel and obvious remedy for their elimination. They are left as historical oddities, to amuse students of logic and clear thinking. The ghost of William of Ockham threatens to haunt academe. With just this one word *other*, the storm clouds over past mathematical panoramas could all be dispersed, and the whole body of set theory freed from paradoxes by being viewed in the light of self-other-reference.

Having expounded his basic revolutionary concepts of Special and General Relativity, Albert Einstein then spent most of his subsequent research exploring the topic of an enigmatic and elusive Unified Field. There are other approaches to this ultimate goal of understanding the Mathematics of the Universe. The themes in the book **ACHIEVING THE IMPOSSIBLE** - *The Quest of Science for the Self of the Cosmos* - of which this thesis is a part, are developed with reference to the Self of the Cosmos as a self-existent selflife essence having life and existence from itself, from its very own *self*, *a se,* and hence called A-se-ity. Aseity's being and becomingness can be expressed comprehensively in terms of a Unified Field of Self-Other Existential Relativity.

GÖDEL'S REPROOF

An ***other*** self-reference paradox in Logic to engage our attention is named after Jules Richard, a French mathematician who formulated it in 1905. It is important, not because it is reminiscent of Russell's Paradox, but for its relevance to Kurt Gödel and his monumental Proof. It is basically just another, though more complex version of the Barber contradiction. It contains words or ideas which are equivalent to, or corresponding with self-defining and not self-defining, self-designating and not self-designating. Richard's argument contemplates defining in a language a function that differs from every function definable in the same language. His logic can be made consistent in its conclusions rather than self-contradictory, by the simple use of the word ***other*** in the context, such that not self-defining now becomes notself defining which means others-defining, and not self-designating becomes notself designating and means others-designating. In other words, **Richard's argument contemplates defining in a language a function that differs from every OTHER function definable in the same language.**

Nothing further is to be gained by elaborating in more detail on Richard's Paradox. It has only been introduced as a lead to the already mentioned work in mathematical logic of Kurt Gödel. In 1931, Gödel, a young mathematician at the University of Vienna published a paper which was later on cited as the most important advance in mathematical logic for a quarter of a century. It was entitled "***On Formally Undecidable Propositions of Principia Mathematica and Related Systems.***" Gödel's Proof, as it is now referred to, is difficult reading even for the best mathematicians and most interested people are simply content just to appraise his conclusions which were revolutionary, to the point of being melancholic or disastrous in their revelation.

Gödel purported to show that the axiomatic method in use from the times of Euclid and proven so efficacious through the Ages, possessed certain weaknesses or limitations even when applied to relatively simple systems like cardinal number arithmetic. There are two main aspects to this lack of completeness, or of self-sufficiency. One is that if any system that includes arithmetical considerations contained a proof of its consistency or freedom from contradiction, then it would seem that it would also contain a proof of its own inconsistency. He showed that it was impossible to demonstrate the

internal consistency of any formal system without using other principles of inference whose own consistency was just as much in question.

The second aspect of self-insufficiency highlights one of the key words of this book. Gödel proved that no logico-mathematical proof in itself is possible for the consistency of a formal system embracing the whole of arithmetic, unless such proof uses other rules of inference that are more powerful or extensive than the actual rules used in deriving the theorems within the system. Other rules, from outside the system are needed in order now to complete or complement the rules within the system. All self-functioning systems are incomplete, and are other-dependent if they are to manifest any kind of positive growth.

For Gödel, no system is truly self-sufficient. Formal deductive systems are proven inadequate and even refute themselves. They are not only incapable of making their own formal deductions consistent, but they are also so limited that they are unable to cope with conclusive statements, which though intuitively true, remain outside the ken of their deductive reasoning.

Gödel's thesis stands out like an end-of-the-road warning light from within the unsettled state of mathematical reasoning, in frustration due to deep-seated irritations with affirmed and negated self-referring teasing paradoxes. His work, though a veritable milestone in the history of the meanderings and vicissitudes of modern mathematical logic, is thankfully not the last word on the subject. His ***Proof*** is not only unnecessary now, but can be shown by linguistic analysis to be intrinsically flawed.

Gödel showed that any system which literally employed terms and mappings involving self and also not self or similar self-contradictory terms or their implicit equivalent, must necessarily end up with its own set of self-contradictions. He came to his novel conclusions, as he himself said, by noting Richard's Paradox and in a way modelling the structure of his Proof on it. Richard came to contradictory conclusions because he took self-contradictory expressions to their logical conclusion. Such expressions were **not self-defining**, **not self-designating**. If he had employed the complementary **notself**, meaning **other-defining**, **other-designating**, instead of the self-contradictory **not self**, consistency

would have been maintained in his logical development. Without realizing the full implications of what he was doing, Gödel proved the necessary use of the word *other* in relation to the incompleteness or self-insufficiency of any formal deductive system. If he had only fed this necessary word back into his own formal system, and applied this word *other*, as mentioned above, in another sense as the complement of self and as synonymous with not-self, he would have avoided all self-contradictions, and the whole business of inconsistency would never ever have arisen.

There is another way of viewing the overall problem of consistency in mathematical logic. The paradox is a literary device and essentially is only a seeming contradiction. A true paradox is such that it can be shown on analysis to contain equivocal or faulty diction. When reinterpreted, the seeming contradiction is removed. The meaning and use of words is the domain of diction and when a person does not know the real meaning of words, or the full import of what one is saying, then faults against diction may result. The contradictions in Set Theory are faults against diction, by not being aware of the double or two-in-one meaning of self-negation's not self, either as a contradictory no self at all, or as one self's complementary other or notself.

A *true* formal or axiomatic system does not have to prove its consistency. Like any scientific theory or model, it can be taken as true until any inconsistency can be demonstrated. Its existence, free from contradiction, makes it truly consistent. It is innocent until it is shown to be guilty. If any contradictions do seem to arise, they are due to one or more of several reasons. There may be faults against diction, inasmuch as we do not really know what we are talking about. There may also be false foundations of inconsistent premises or erroneous postulates. There may be some invalid use of the rules of logical inference. Mathematics, as a challenge to human intelligence, is an evolutionary game that must be played to an ordered and logically consistent set of rules. At times these will need the addition of other rules with a continual examination and reinterpretation as new horizons come into view. Unsolved problems remain an uncompromising challenge to the ever-questioning human mind seeking for rational formalistic solutions in its left-lobed brain for the novel intuitions in contemplation of its right-lobed

counterpart or complement. As with some relations with Prime Numbers, there are mathematical *truths* which our intuition accepts as logically valid and consistent but which elude deductive proof in axiomatic systems.

Given his own terms of reference within a specified context like ***Principia Mathematica***, Gödel's fundamental concepts of self-insufficiency and of other-dependence are quite valid logically. The formal method of his monumental Proof may stay as an historical curiosity, but in any comprehensive reappraisal of his thesis and its import where cognizance is taken of all self-other-reference operations, it must now be deemed as no longer relevant.

The one simple word ***other*** makes unnecessary the millions of other words used with only partial success in the past trying to free Modern Set Theory and Mathematical Logic from illusory contradictions. In the acceptance of the self-other nature of all positive growth in knowledge, there is guaranteed to formal systematic learning a complete security from all possible inconsistencies. There is also guaranteed a rational solution to the problems of self-reference and algorithmic computability as found, for example, in the applications of Set Theory to recursive functions and the like, in Turing Machines and Computers, where the universal **ALL** must be understood as the union of **SELF** and **OTHER** if both reflexive and transitive relations are to be contemplated together. As in physical science, the interaction of the observing self and the observed other must always be taken into account, the experimenter being an integral part of the experiment, so also in Set Theory's study of mathematical relations, the existential relativity of all self and other knowledge must be born in mind. Sets only exist in the mind of a self who identifies its one reflexive self transitively with some other or all other or no other in the unity of the psychical becomingness of knowledge. Modern Set Theory accepts the distinction of reflexive, symmetric and transitive relations among the units of sets, but the reflexive and transitive relations of knower subject and known object have their own special meaning in any systematic theory of knowledge or Epistemology.

LIARS AND LIES

Contrary again to Russell's facetious epigram quoted earlier, we do need to believe that what we are honestly saying is true, i.e., that it is consistent and free, we hope, from all possible contradiction. There are some further even more fundamental self-reference logic paradoxes whose elucidation requires that we know, not only what we are talking about, but also whether what we are saying is true.

Consider the statement, "This statement is not true". We are asked "Is it true or not?" There are several aspects to the *true* answering of this question which, arising from a dishonest statement, is really not an honest question. Logicians who assert that this statement is quite meaningful should be prepared to define what they mean by *meaningful* and *not true*, and state what really constitutes a *true statement*. This they seem reluctant to do, but prefer to bandy the word *true* around like any other cheap four-lettered expletive. People who ask whether their confessedly *not true* statement, i.e., their self-confessed *lie*, is true betray ignorance of what a true statement is. Persons who ask whether a *not square* figure is square are generally ignored by logicians, while those who ask whether an intentionally expressed *not true* statement is true are considered by some as novel thinkers and initiators of new thought processes. How does the asking whether a not-square figure is square, differ from the asking whether a not-true statement is true? Both *It is true...* and *It is not-true...* statements involve some sort of reflexive self-reference to *true*.

Philosophers may dispute about the nature of *the true* or ask "What is truth?" However, in the everyday acceptance of the meaning of *true*, statements which can be preceded by "It is true that ..." or "It is not true that..." are taken as valid propositions for logic-processing and are named Truth Functions. There is some concern for the intellectual honesty and sincerity of those logicians who postulate Truth Functions, whilst at the same time they are reluctant to give any precision to what they are talking about when they use the word *true* or *truth.*

A simple statement such as, "This statement is not true", or "This statement is a lie", cannot be accepted as a valid truth function since it has already prejudged its acceptability and decided its material logic-court's sentence of guilt with its own self-condemnation. A

statement that repudiates all acquaintance with truth cannot ever expect to be invited to any Truth Table Function.

If such an invitation is extended and accepted, then the statement must change its name now from *This statement* to *That statement*, or to *This other statement.* In which case, Truth Functional Analysis is now concerned with the duality, "It is true that *That* (*This other*) *statement is not true*", or "It is not true that *That* (*This other*) *statement is not true*". In neither case is there any internal material logic contradiction, and the self-deceptive dishonesty merits no further useful consideration in academic circles.

Statements like, "This circle is a square", or "This barber shaves those who do not shave", are self-contradictory in their very meaning, and have no place in any serious rational system of thought. Russell's epigram is a half-truth. In Mathematical Logic, we must know at least whether our premises, assumptions, postulates or whatever we are starting with are consistent or not consistent in themselves. We must know whether they are implicitly or explicitly self-contradictory and know also the meaning of what we are talking about. We must know what a statement means in itself, in its self, i.e., we must know what the self intends who utters it. Over twenty-five centuries of the study of Logic have made this discipline precise and consistent. Our reasoning processes are virtually foolproof. If inconsistent conclusions arise, they are due to inconsistent or self-contradictory premises.

In statements such as "I am a liar" or "This statement is not true" we may ask what these statements mean in themselves, in the self who identifies its self with them and then utters them. What does some self have in mind or intend who says, "I am a liar" or "This statement is not true". Real becomingness implies positive growth. An honest self's witness to its own integral and consistent other-becomingness is true, is truth. People, who with all honesty try to say what they mean, believe what they are saying to be true. Consistent means free from contradiction, and a statement contains a contradiction if it asserts something both *to be* and *not to be* at the same time under the same circumstances. A geometrical figure cannot both be a square and not a square at the same time under the same circumstances.

LIARS AND LIES

As in Courts of Law, all statements made by a defending self are to be accepted as innocently true. To publicly confess as to their being guiltily not true is to anticipate judgment and subsequent sentence of condemnation. Assuming that we are dealing with people of integrity, all statements uttered by them are to be taken as true in themselves, in their selves. It is only in the subsequent judgment of the same selves and others that they become eligible for consideration as Truth functions. In these cases as pointed out, the "*This statement*" should now become a "*That statement*" or a "*This other statement*". Statements mean what the self intends who utters them. Others may indeed question whether a statement is true or not for them, but in itself, an honest statement says what its honest self means and means what its honest self says, regardless of its material logic.

No "*This statement*" in itself, in its self, can be literally both *true* and *not true* at the same time. In their honest selves, all statements are intentionally true and only with respect to other selves can their material logic be called into question. With "I am a liar", the statement may be true, if I mean that it is only all **other** statements that I make that may or may not be true. It becomes self-contradictory if it implicitly says "I am not a liar when I say I am a liar". If I intend my self-accusation of dishonesty to extend to the very statement itself "I am a liar", then I become a father of lies. I forfeit all credibility and my self-confessed lack of verbal integrity excludes me from all further meaningful dialogue.

The situation with the statement "This statement is not true" is such that in its self, it is self-contradictory. It implicitly says, "This true statement is not true", and hence must be placed in the same category as all other absurdities like, "This circle is a square", and banished from further honest and truth-seeking logical discussion with its quest for positive and consistent becomingness.

There are other seeming contradictions, apart from those directly associated with Russell's Barber, which arise because persons do not make the necessary distinction between **this** and **that.** For example,
Sentence A. *This sentence does contain seven words*. Not true.
Sentence B. *This sentence does not contain seven words*. Not true.
Sentence C. *That sentence* (A) *does not contain seven words*. True.

LIARS AND LIES

If we are told that since A is not true, then its mere negation should be true, then we must understand that the negation of A is C and not B. Such **this-that** dualities are another aspect of **self-other** complementarity and have undertones of Russell's original not self-referring antonym.

Logicians have reduced the famous Plato-Socrates paradox to the simple form,

Let A be the statement - *B is not true.*

Let B be the statement - *A is true.*

Statements A and B may be subjected together to a proper Truth-Table analysis with respect to their **If-then** material implication being true or false. If the simple material implication of formal propositional logic is all that is in question, there is no contradiction. If however we seek some connection, some kind of causal interrelation between A and B, then they cannot both be true. The statements can only remain as they stand and their formal logic free from contradiction when both A and B are false. In this case, Plato and Socrates were both wrong.

NEO-CANTORIAN S.T.U.F.F.

Whilst modern formal axiomatic set theory is not disposed to any further involvement with paradoxes arising from self-reference situations, the latter still figure prominently in general text books of Set Theory and in the writings of some authors of best selling non-fiction scientific and philosophical works. Most comprehensive encyclopædias and books dealing with the history of Mathematics and expositions of its principal ideas devote considerable space to Russell's Barber and the past inability of logicians to come to terms of *self* and-or *other* reference with what Ockham's Razor would now deem as really only seeming and finally removable contradictions.

Last century saw the development of non-Euclidean geometries which were more adapted to the mathematical representation of curved space in the Cosmos. This century has seen their further evolution as well as the formalizing of various set theories which aimed to make consistent what Cantor's algebra of sets had initiated. Russell's basic paradox in the foundations of Set Theory can be truly eliminated, not merely circumvented, by distinguishing between ordinary sets of sets which contain other sets only and those extraordinary sets of sets that contain both themselves as well as other sets. The accepted Z-F and N-B-G set theories, whilst consistent and adequate in their restricted context, are not applicable to the much broader spectrum of the self-reference sets of self-functioning- feedback-systems.

Conceptually a factual system may be understood and described as a set of distinct procedural parts which are made to work in union now as a unity or whole for some common purpose. In encountering self-functioning-feedback-systems we have been forced to face the reality of systems, the sets of whose parts contain the very systems themselves as true proper subsets of themselves. As observed earlier, sets that do not contain themselves have been termed *ordinary* by some logicians, whilst those that do have been classified as *extraordinary.*

For a steam engine with a speed-governor, the working whole is as much a part of the system as are the other parts themselves. The set of parts for such a self-functioning system contains not only the distinct units but also their working union. The real unity of every s-f-f-s results from the union of distinction's *distinction* and *union.* The non-living speed-governed system is self-functioning because itself's

wholeness totally partakes of the nature of another self, the artificer self who has dictated the new artifact's existence, and made it to be what it is. It is the extension or becomingness of such a self's being. It is a projection of the psychical self into inanimate physical matter.

The concept of a set that contains itself as a member of itself is intriguing and merits further exploration. Actually, such a set must be a set of sets. Indeed more, it must be a set of sets of sets...ad infinitum. Let Y η {All other sets}. Let X η{X, Y}. Wherever we find X, we can replace it with that with which it is identified, and then X η { { { {...,Y}, Y}, Y}, Y}. This can go on forever. A set that contains itself as an element or member must be a unique kind of infinite set, in the sense of being capable of endless substitution, and of consequent infinite evolutionary self-imaging expansion within its own self.

A most interesting and instructive aspect of self-mapping or self-substitution is in the phenomena of fractals, as with the Mandelbrot Sets. Using simple iterative self-other-containing sequences of complex numbers, the most extraordinary ordered patterns can be generated on a computer screen. What is even more extraordinary is that any small part of such fractal artistry can be subjected in turn to similar limitless iterative magnifying, and each newly resolved enlargement continues to exhibit a unique and astonishing beauty and complexity.

In a somewhat different context we may say that the set of all infinite sets is itself an infinite set, and hence contains itself as one of its own elements. Where do infinite sets begin and end? They all obey the LOGOS Rule and exist in the infinite Self-Other-Life-Set of Aseity whose ovoid dual-focal reflecting nature is the archetypal Alpha and Omega of all self-other-functioning-feedback-systems of existential relativity in the Cosmos.

In the evolutionary developing of their knowledge, Mathematicians have made use of special signs, called *operators*, to help in their understanding and abstraction of the dimensions of this space timed Cosmos. Most people are familiar with the Greek π, as the sign of the ratio of the length measures of the circumference of a circle to its diameter. If c stands for the length measure of the circumference of a

circle and r for the length measure of its radius, then $c = 2\pi r$. Popularly, π is taken to be about $3^1/_7$ or approximately 3.142. It is

neither a recurring decimal, nor can it be expressed as a finite nonrecurring decimal. Not being an algebraic number, which only exist as solutions of algebraic equations with integral coefficients, it is also designated as a transcendental real number. It can be expressed as the sum of an infinite series, as can the circular or periodic functions of sine, cosine and tangent, and also the base e of the exponential growth function e^x. This latter operator e is transcendental too, and can be expressed approximately as 2.71828.

There is a hidden mystique about the exponential function e^x. It could well be called an extraordinary self-function, being the base of all self-functioning growth. The rate of change of e^x with respect to the variable x is e^x itself. The whole function e^x is always the existential and essential part of all its derivatives and integrals. It is uniquely self-containing and is self-mapped into all its associated analysis processes of evolution and involution.

Of special interest to Chaos Theory is the witness observational science bears to the existence of kinds of self-other-functioning-feedback-systems, both natural and artificial. Electromagnetic radiation, in its spaced time propagation, reveals duality's complex existential relativity. All imaging complementarity in Physics as with fundamental particles evinces this, and Biology's Genetics bears also a similar witness in the sexistential relativity of chromosomal distinction and union.

Sets are composed of distinct elements or members or units. Sets have subsets. The set of vowels and the set of consonants are both subsets of the set of letters of the alphabet. In their listing of all the possible subsets of a set, past logicians have included the latter set itself, but have termed it an improper subset of itself. All the other possible subsets are called proper subsets. This author's Neo-Cantorian revision of Set Theory, whilst retaining most of its generalizations, rejects the accepted notion of an improper subset along with such prevailing concepts as the unit and null sets.

A set, as a well-defined unity of distinct units in union, is both singular and plural at the same time, a collective noun that enjoys the unity of the continuum's union with the distinction of discrete units.

as Ø. We rightly distinguish between the three distinct letters of the alphabet a, b, c, and their union in the set {a, b, c}. In this revision, supposed sets such as {a} and { } are meaningless and unacceptable since, with only one member or no members at all, they contradict the definition of a set as a well-defined collection (singular) of distinct entities (plural). What is called a unit set in other theories is understood here as only a unit, not a set. Every set itself is a unit set and is a unity of units in union, and *units* is grammatically plural. Equations have either no solution, or one solution, or a set of solutions. .

The cardinality of a s-f-f-s is always one more than that of the set of the parts of the not self-functioning system constituted by the not self-functioning parts on their own without their self-functioning union as a whole. Differentiation into mere parts requires their added integration into a *wholesome* union in order to become a real s-f-f-s. Mere differentiation results in listing and numbering only the accumulated dimensions of discrete parts whilst integration adds a totally new dimension as their continuum.

The human mind has struggled from the dawn of self-consciousness with the idea of the never-ending, of that which goes on for ever and ever. From the days when Zeno deliberately confused the Athenians with space-race paradoxes, right up until the present time, notions about *the infinite* have teased thinking people with situations of how apparently logical reasoning could lead to evident absurdities and contradictions. What is *infinity*? As various techniques for counting improved, so numbers were invented for bigger and bigger collections of things. Finally, it was realised that there were no limits to human thought and that no matter how big a collection was, it could always be added to in theory. A set is said to be infinite if it contains as many elements or units in its whole as in some fractional part of itself. **Infinity is not numerical bigness, but a self's inexhaustible self-other subsetting divisibility, a self's limitless iterative distinctioning of units in union in the unity of self-other existential relativity.**

With the introduction of ideas of cardinality and equivalence in Set Theory, new horizons or modes of thought were revealed for a

better understanding of such past problems concerning infinite sets of things in particular finite situations, like points on a real number

line. The cardinal numbers of finite sets were called natural numbers.

Cantor called the cardinal numbers of infinite sets *transfinite cardinal numbers* or *transfinite powers.* Cardinal numbers, whether finite or transfinite, are accepted measures of the how-many-ness of the units in union in the unity of a set. It is unimportant just how a cardinal number is in fact defined. It is a very convenient device to relate the real number-experience aspect of sets when their units or elements are put in an ordered one-to-one correspondence with each other. If the matching process is perfect, the two sets are equivalent and they are said to have the same cardinality. By defining equivalent sets in terms of the matching conjugation of their respective units, equivalence is formulated independently of finiteness or transfiniteness.

The set of natural numbers is equivalent to its subsets of both the even and odd numbers and they are all thus classified as infinite sets with the cardinality of the transfinite number $\pm o$. The set of all rational numbers has the cardinality of $\pm o$. The power set of all possible combinations or sets of these rational numbers has a different cardinality $2^{\pm o}$ or *c*. This new transfinite cardinal *c*, called *the power of the continuum* signs the cardinality of the set of real numbers or of the latter set's real-point-matching, one-to-one correspondence on a number line. A continuum of real numbered points generates a real numbered line. Likewise, a continuum of real numbered lines generates a real numbered plane, and a continuum of real numbered planes in turn generates a real numbered space. It can be shown that the cardinality of real numbered points in a real numbered space is still *c*.

Cantor's so-called Theorem can be understood as expressing the obvious fact that the cardinality of any set A is always less than the cardinality of the power set of A which is the set of all the possible subsets of A. As explained above, □o is less than c, the latter c being equal to 2□o. How far can this power-set generating proceed? What if A is already a power set of sets? Cantor conceived an unending hierarchy of transfinite cardinals applying to an unending sequence of new power sets of power sets...ad infinitum.

The trouble is with the word all and with not distinctioning between reflexive and transitive cognitive relations. The satisfactory resolution of the difficulties inherent in Cantor's Theory of Sets can

be had by consideration of the LOGOS rule, where self's *all* is the unity effected by the union of distinction's reflexive self and transitive all other. If X stands for the Set of All Sets including itself, and Y for the Set of All Other Sets, i.e. other than X, then we can understand that as in a s-f-f-s, X η { X, Y }. In a set of sets, each set is a unit set in union with other unit sets to conceive a set-unity. X and Y are the two primary or archetypal unit sets whose distinction in union biunifies the unity of the unit Set of All Sets X, whose individual reflexive cardinality as of the set of points on a real number line is $2^{\pm o}$ or *c*. Transitively with respect to X, Y is a unit set whose cardinality is *c* - 1. There can be understood then a transfinite cardinal number between $\pm o$ and *c*. It is *c* - 1, the cardinality of the unit set of all other unit sets in the unity of the unique set of all sets. The latter's inclusion of itself, of its self, is dictated by its dictator-self whose being is biunified with its known *other's* becoming, in the existential relativity of all knowledge. In the appropriate context, when *self* says "*all*", it means both its *self* and *all other*.

Whilst rejecting the use of Ø in connection with the hypothetical null set, it is convenient to retain Ø as the sign of the self's concept of the *no other.* As representing the psychical entity *no other unit*, it may be understood as being the implicit last unit of every ordered set. At the core of all self-mapping, or of any reflexive or self-reference relation, is the axiom that in any ordered set of sets, the first unit set is {self, no otherself}, {self, Ø}. The self of this set is the self who knowingly names it by identifying its own self's being with it in the becomingness of all knowledge. A set is only verifiably void or truly empty, like an empty room, when there is no one present in it other than the observer's own one self. In Self-Other Existential Relativity, this first self-unity defines the unit empty or void set and anticipates the set of binary arithmetic's assigned numerals, {one, no other-one}, {one, none}, {1, 0}. Forms of the latter are the basis of all digital computer technology and exemplify the concept of *creatio ex nihilo*, creation from nothing.

The old abstraction of a metamathematical Set of All Sets, a Unity of All Unities, is made more intellectually digestible in the light of

the above revision of Neo-Cantorian Set Theory. There is a place in Mathematical Logic for the Set of All Sets, with its special inclusion of those extraordinary sets which include themselves. It is time such

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Set of All Sets, such Field Unity of All Unities, was brought back from its enforced exile and burial in The Archives of the History of Mathematics. This Unity of All Unities presupposes the distinction and union of distinction and union themselves in unity. Not merely should it be resurrected, but it should be enthroned, along with the LOGOS rule, as the two unifying concepts which make possible all other knowledge expressed in both particular and universal ideas.

Set Theory is not just a unifying concept in a formal system of Mathematics. It is much more. Its laboured birth-pains, though annoying and seemingly superficial and trivial, are not mere surface problems but reach far deeper into the very nature of cosmic unity and the self's thought processes. This revision disposes of the troublesome paradoxes of self-reference and not self-reference in Set Theory with the bonus of initiating new outlooks for aseistic cultural evolution through the introduction of a new simple logic of the extraordinary.

A consistent Theory of Everything must embrace concepts of both Unity and Infinity. By defining a set as a *Unity of distinct Units in Union*, a meaningful Philosophy of scientific and mathematical distinction and union can be initiated. Any restricted formal axiomatic set theory in which *Set* and *Set Membership* are undefined terms would be more appropriately called Container Theory. With such, it is meaningful to ask how many things are contained in a container and to receive the answers, none or one or more. Existential self-other Relativity does not admit a Set-Unity of emptiness or of one thing only.

We are tempted to think of the braces, i.e., the curly brackets in Set Theory, as being a kind of magical container. There is a danger in giving this or any kind of quasi-material existence to sets. Sets only exist in a mind which reverses the spaced time physical distinction of fission's explosively breaking one whole into numerous parts, by fusion's implosively integrating the many differentiated parts into one whole or universal idea. The distinct units in union in the unity of a set may be physical or psychical entities. The rivers of the world have their own separate physical existence. The set of all

such rivers exists psychically as a new whole entity in the sensibly fertilized mind of a self who conceives this unity or universal idea.

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We find it easy to ridicule the later scholastic mind which speculated on the number of angels who could position themselves on the point of a pin, but the problem is akin to the mathematician's enquiry into the number of discrete positional points in a continuous line. Our theory of the mathematics of infinity or of transfinite numbers has developed from such problems as the non-countability of points between other points or the number of distinctioning cuts that can be made in a continuous join or union of two points.

Set Theory need no longer fear paradoxes. The enigmatic Set is both a **one** and a **many.** It is a many-in-one or polyunity. It is a unity of units in union which is what universal ideas are all about. The Set of All Sets is the Unity of All Unities. The psychical order reverses as well as reflects the physical. All knowledge, like complex number relations, follows a set pattern, the union of an alpha or I-self with its co-focal omega or i-image otherself in the unity of the ovoidal psyche. Meditation or rational reductionist analysis is concerned with the units, contemplation or intuitive holistic synthesis with the unity. In the continuum or unity of the one self there is scope for an infinity of infinite diversities. One's spaced time self is both point and line, both surface and volume, both continuous wave and discrete particle. In the existential relation of self-other knowledge, the knowing self is able to comprehend the existential relativity of the radiation phenomena of the self-other-propagation of complex electric and magnetic fields. In a spaced time, it can apprehend light, not only as a measurable wave-entity in terms of frequency and wave length, but can also perceive it as a continuum's set of particularized photons. A physically informed, yet essentially spaceless, and hence timeless psychical self, can identify itself with a never ending matrix of space-points and time-moments, each separately or all together. Spaced *here-there* and timed *now-then* exist meditatively in the mind of the self who perceives them. The awareness of the unity of the ever present *here-now*, gives the unique thrill to contemplation's experience of oneness.

There is another recently unveiled phenomenon in the domain of Physics which is completely different from fractals and throws

added light on some whole-part, unity-in-diversity, set relationships in the Cosmos. It goes under the name of Holography and is a system of photographic storage which does not use the refraction properties

of lenses but rather is effected when the wave field of light scattered by an object is recorded on a photographic plate as a diffraction or interference pattern.

Though the mathematics of holography had been worked out by Dennis Gabor in 1947 and subsequently earned him a Nobel prize in 1971, it was not until the laser was invented that his theory could be satisfactorily demonstrated. Since then, holography has brought about a most profound Paradigm Shift or new perspective of reality.

The photographic recording of the image is called a hologram. The latter appears as a meaningless pattern of whorls. When the hologram is placed in a coherent beam of light like a laser, the original wave pattern is regenerated as a three-dimensional image. The fascinating reality of a hologram is the fact that even the smallest piece of it, when taken separately and enlarged to the size of the original plate will reconstruct the entire image and not appear just as an enlargement of a part. The part is in the whole and the whole is in each part**.** Such an intriguing situation when each part has access to the whole provides speculative approaches to topics as diverse as brain-functioning and religious mysticism.

A holographic photo can be taken of a donkey so that its image fills the whole picture. If now the corner section comprising just the donkey's head is cut off and this small section then enlarged to the size of the original photo, you will not get a picture of just the enlarged head of a donkey but a repeated picture of the original WHOLE donkey. Each separate part of the picture contains the whole picture in an *implicate* form. The part is in the whole and the whole is in each part. Such an intriguing situation when each part has access to the whole is opening up completely new horizons for the understanding of phenomena in both the physical and psychical realms and which are as far apart as life and death.

There is much more work for Ockham's Razor. The study of self-functioning-feedback-systems (s-f-f-s) in the new discipline of Aseistics and the development of an elementary Neo-Cantorian Set Theory Underlying Feedback Functions, acronymically termed *STUFF*, rationalize conclusions which controvert many cherished

traditions in other branches of learning. Within STUFF's revised terminology, every real s-f-f-s, being a set which contains itself as a proper subset of itself, is mathematically infinite and extraordinary.